



Research Article

## An Improved Non-dominated Sorting Method in Genetic Algorithm for Bi-objective Problems

Mirpouya Mirmozaffari\*

*Department of Industrial Engineering, Faculty of Engineering, Dalhousie University, Halifax, NS, Canada*

### Keywords

Crowding distance,  
meta-heuristic algorithm  
NSGA2.

### Abstract

This paper proposes a new non-dominated-sorting method for non-dominated sorting genetic algorithm (NSGA2). This method combines crowding distance and distance-based methods for non-dominated-sorting. The distance-based method identifies concavities on the Pareto Front curve and selects the solutions located at these points. Our goal in this method is to converge faster and find more optimal solutions in the Genetic Algorithm. The Numerical example shows that the results obtained from NSGA2 method with modified non-dominated sorting are better than regular NSGA2.

### 1. Introduction

The first concept that programmers or software developers deal with, is the concept of algorithm. These algorithms are one of the major fields of artificial intelligence in solving real world problems. One of the simplest problem-solving techniques in the field of artificial intelligence is evolutionary computation methods. In general, evolutionary computation algorithms are based on Darwin's Theory of Evolution align with the implementation of computer programs. Evolutionary computation algorithms are one of the problem-solving methods which are based on population and trial and error and benefit from the stochastic optimization's mechanisms or meta-heuristic optimization's mechanisms to find global optimal solution or approximate optimal solution.

One of the initial meta-evolutionary algorithms is the annealing algorithm [1]. This algorithm is generally based on the similarity between the cooling of molten solids and combinational optimization problems. Another algorithm Inspired by nature is the artificial immune system algorithm [2]. In general, artificial immune system algorithm is inspired of biology science. The artificial immune system is a model for machine learning. The ant colony optimization algorithm proposed by Dorigo in 1991. It is used in solving the traveling salesman problem and multidimensional allocation problems. The ant colony optimization algorithm benefit from simple factors called ants to generate solutions. Bee cloning algorithm was introduced in 2007 by Karabog and Bashturg [4]. Bee colony algorithm examines each point in the parametric space, consisting of possible responses as a food

\* Corresponding Author: Mirpouya Mirmozaffari  
E-mail address: [mr828394@dal.ca](mailto:mr828394@dal.ca)

Received: 10 June 2021; Revised: 27 June 2021; Accepted: 3 July 2021

source. Watch bees randomly simplify the solution space and report the quality of the visited positions using the functional competency [17-22].

In many papers, evolutionary operators are used in a modified form. Modified operators will be able to generate better solutions for particular problem [23-29]. By designing three mutation operators in his algorithm, Leyla Sadat Tavassoli was able to obtain more uniform solutions in a wider range [5]. She also managed to get more accurate answers by adding error functions to the Genetic Algorithm. Susmita presents a complex algorithm for his mutation operator. This operator classifies populations and divides them into several groups before generating solutions, and then generates a set of solutions for each group according to the characteristics of that group [6]. Salvador used Modified crossover operators based on value closeness in his article. The proposed crossover based on closeness is designed to operate only for thresholding problems [24]. It takes two particles typically know as parents to generate two offspring [7]. Moreover, Mahjoob design and modified operators of heuristics algorithm to generate better solutions for routing problem [8, 9]. His works has significant impact to reduce fuel consumption of supply chain systems.

In recent years, much research has been conducted to improve evolutionary algorithms. One of the most important is to create an algorithm with improved operators to create new population in other word offspring's population. Saber offers two operators, Non-uniform mutation and Polynomial mutation, to generate offspring's population [10]. These operators can generate solution around parents. In these operators the probability of mutating a solution near to the parent is higher than the probability of mutating one distant from it. In addition, Ryoji has conducted research to improve Exponential Crossover operators in evolutionary algorithms [11]. In his paper, Ruihua promotes nondominated-sorting for non-dominated sorting genetic algorithm (NSGA2) [12].

Other than optimization by evolutionary algorithms, in many papers new optimization methods have generated solutions [13-15].

In this paper, we present a new nondominated-sorting method to generate better solutions. This method prevents to elimination of solution located at critical points by identifying these points on the pareto front curve. Therefore, the convergence speed of algorithm will increase and better solutions will be generated.

## 2. Problem

In Genetic Algorithm, a new population is created after the offspring are generated and combined with the original population. To select the final solutions, these solutions enter the fitness algorithm and are classified there. This classification is based on dominated-sorting and nondominated-sorting. Solutions which are in a same group, are classified based on nondominated sorting. In the NSGA2, nondominated-sorting is performed based on crowding distance. In this method, the minimum distance of a certain solution compared to other solutions are considered, and each solution which has a greater minimum distance compare with other solutions is higher fitness.

One of the disadvantages of crowding distance method is in finding critical solutions. The critical solutions are located on convexities and concavities of Pareto front. These specific solutions are higher fitness in classical optimization methods such as weighed sum. To understand more in depth, let us to provide an example. Figure 1 shows a bi-objective problem plot containing 10 solutions. These 10 solutions belong to the pareto front and are all in the same level in terms of domination. In crowding distance method, due to the minimum distance to other solutions of solution number 7 is lower than that of solution number 6, solution number 7 is in the risk of elimination.

If we connect all 10 solutions by a line, a conviction will be created in the solution number 7 area. Each solution, such as solution number 7, which is placed on a conviction, have a higher fitness in classical optimization methods compare with other solutions. To preserve these items, a method to identify these solutions is needed.

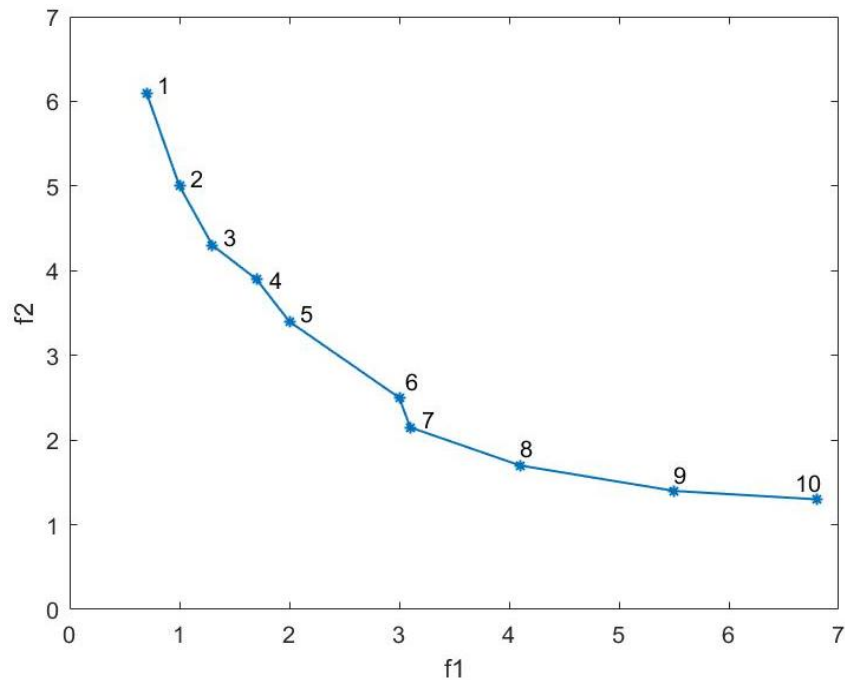


Figure 1. Bi-objective problem plot containing 10 solutions.

### 3. Methodology

The method we proposed in this paper to find pareto front concavities is distance from vertical line method or distance-based method. In this method, for each solution which are in the same level, except the first and last solutions, previous and next solution are connected by a line and distance of the middle solution to that line is measured. These distances are identified for solutions that are on the same level in terms of domination. After calculating absolute value of distance, distance sign will be determined.

To determine the sign, we benefit from line slope formula. As shown in Figure 1, the slope of line between solution number 6 and 7 is greater than the slope of line between solution number 5 and 6. Thus Solution number 7 is placed on a convexity. For those problems which are bi-objective and minimization problems, we consider for the calculated distance a negative sign for convexity and a positive sign for concavity. For problems where both objective functions need to be maximized, for a convexity a positive sign, and for a concavity a negative sign are considered. At the end, each distance has a greater value, has a higher fitness.

### 4. Solution

In NSGA2, sorting of solutions is based on dominated sorting. After that, for those solutions which are in a same level, nondominated-sorting is classified solutions. In NSGA2 nondominated-sorting operator is crowding distance. The proposed method in this paper for nondominated-sorting, is a combination of distance-based and crowding distance method. *In our proposed algorithm, two methods run simultaneously, and it is possible to benefit from advantages of both methods at the same time to find the optimal solutions.* As shown in Figure 2, by determining a value of  $\alpha$ , the effectiveness of either of the two methods in nondominated-sorting is identified. The value of  $\alpha$  is between 0 and 1, which in case  $\alpha$  is closer to 0, more solutions

will be generated by crowding distance method and if closer to 1, more solutions will be generated by distance-based method. Therefore, if value of  $\alpha$  determine 0.5, solutions generated from two methods are equal.

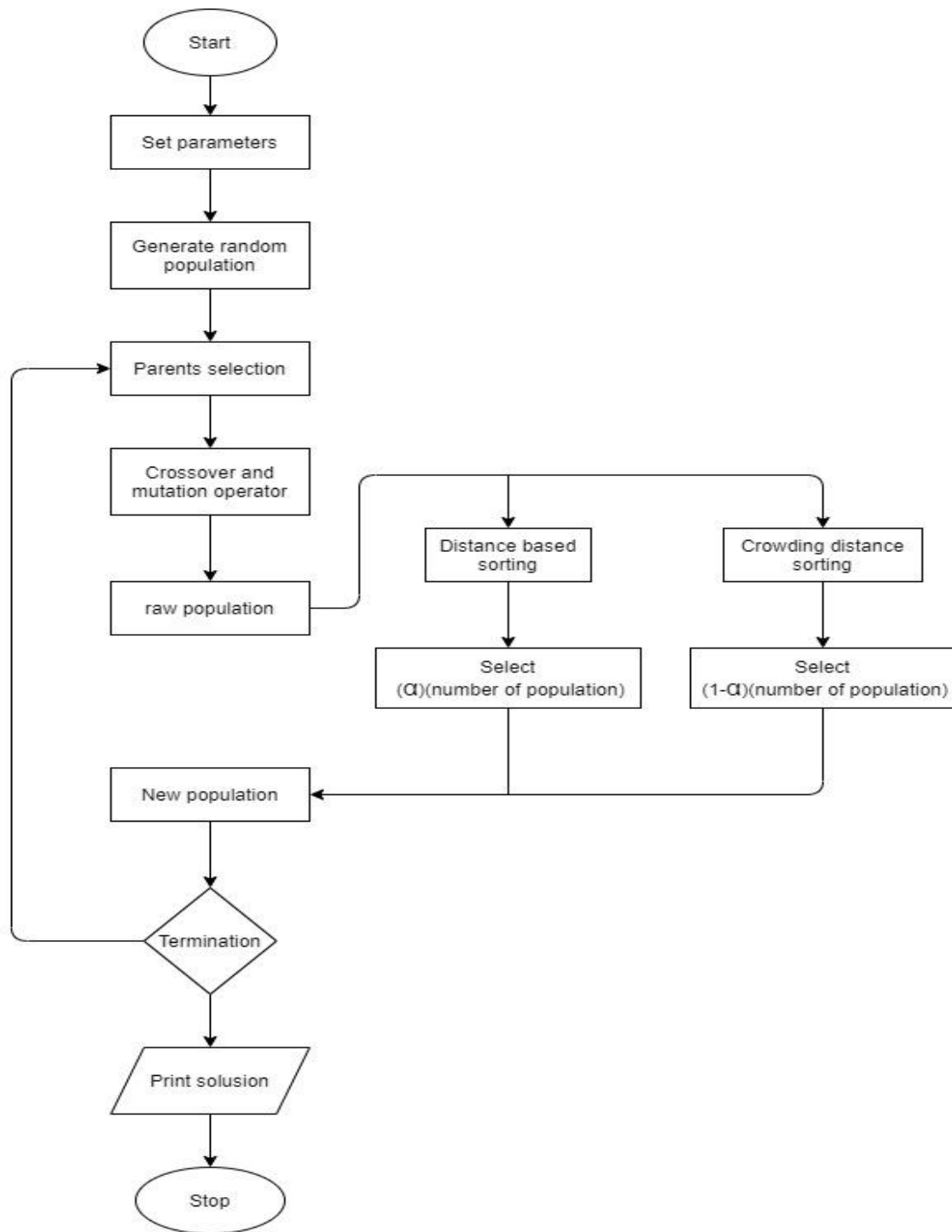


Figure 2. Proposed method.

## 5. Result and Discussion

To evaluate performance of new method, an optimization problem is needed. For this purpose, an optimization problem has been selected from the book evolutionary algorithms for solving multi-objective problems to evaluate performance of the new method compare with regular NSGA2 [16]. This problem comprises two objective functions (1), (2) and two constraints.

$$f_1(x) = 1 - \exp\left(-\sum_{i=1}^n \left(x_i - \frac{1}{\sqrt{n}}\right)^2\right) \tag{1}$$

$$f_2(x) = 1 - \exp\left(-\sum_{i=1}^n \left(x_i + \frac{1}{\sqrt{n}}\right)^2\right) \tag{2}$$

s.t.  $-4 \leq x_i \leq 4$ ;  $i = 1,2,3$

This problem is performed for different values of  $\alpha$  by NSGA2 with modified non-dominated sorting operator. This problem is performed for  $\alpha = 0, 0.25, 0.5, 0.75, 1$  and obviously in case  $\alpha$  is equal to 0, this algorithm will be similar to regular NSGA2. In addition, since a reference is needed to evaluate performance of the algorithm, this reference is created by selecting 100 of the best of all  $\alpha$  values solutions for all generations. This experiment has been conducted by a MATLAB software version R2020a. To run this program, a computer with i7 CPU specifications, 1.8GHs and 16 GB of RA was used.

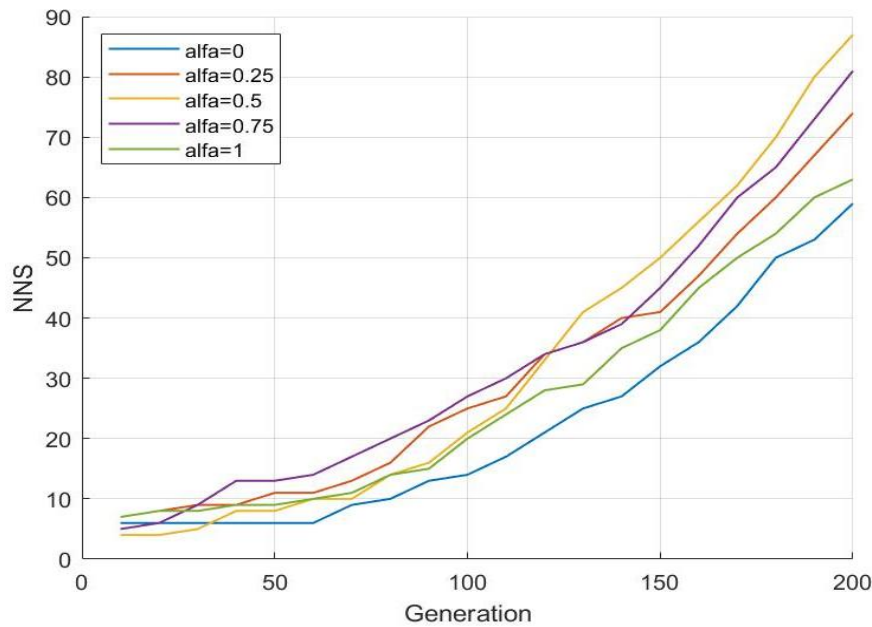


Figure 3. NNS for 5 different  $\alpha$  values from first generation to generation 200.

One of the best criteria for comparing meta-heuristic algorithms is to compare the number of non-dominated solutions (NNS) by reference. Figure 3 shows NNS for 5 different  $\alpha$  values from first generation to generation 200. As shown in this graph, NNS has the lowest value when  $\alpha$  is equal to 0 or 1 in most generations. This indicates that if generated solutions in the algorithm are a combination of distance-based non-dominated sorting method and crowding distance non-dominated sorting method, the algorithm can generate more optimal solutions.

Table 1.  $\alpha$  and  $\Delta$  values

$\alpha$	$\Delta$
1	0.607
0.75	0.530
0.5	0.464
0.25	0.563
0	0.428

In Delta method, the uniformity of distribution of solutions in Pareto front are evaluated, which is calculated from the below equation.

$$\Delta(s) = \sum_{i=1}^{|s|-1} \frac{|d_i - \bar{d}|}{|s|-1} \quad (3)$$

In Eq. (3),  $d_i$  is equal to Euclidean distance between two consecutive solutions from Pareto front with S number of solutions and  $\bar{d}$  is equal to average of  $d_i$ . The lower value of  $\Delta$ , the more uniform distribution for pareto front solutions. As shown in Table 1, the reason for the greater delta value at smaller values of  $\alpha$  is using crowding distance operator. Generally, crowding distance operator generate more uniform solutions and eliminates solutions that are very close to other solutions compare with non-dominated sorting distance-based operator.

## 6. Conclusion

In this paper, we proposed a new method for non-dominated sorting in NSGA2 algorithm. This method, which is a combination of crowding distance method and distance-based method for nondominated sorting, identifies concavities in pareto front and presents a new method for non-dominated sorting. Combination both crowding distance and distance-based methods in this algorithm leads to find more optimal solutions. Finally, with representing an optimization problem, new method generates more optimal solutions compare with regular NSGA2 algorithm. In our future works, we intend to improve nondominated Sorting Genetic Algorithm for three objective Problems using distance-based method for nondominated sorting.

**Conflict of interest:** Author declares no conflict of interest.

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